

13.12 Given the following functions $F(s)$, find $f(t)$. **CS**

$$(a) \quad F(s) = \frac{s+1}{s(s+2)(s+3)}$$

$$(b) \quad F(s) = \frac{s^2 + s + 1}{s(s+1)(s+2)}$$

SOLUTION:

$$a) \quad F(s) = \frac{K_1}{s} + \frac{K_2}{s+2} + \frac{K_3}{s+3} \quad \left\{ \begin{array}{l} K_1 = \frac{1}{(2)(3)} = 1/6 \\ K_2 = \frac{-2+1}{-2(1)} = 1/2 = 3/6 \\ K_3 = \frac{-3+1}{-3(-3+2)} = -2/3 = -4/6 \end{array} \right.$$

$$F(s) = \frac{1}{6} \left[\frac{1}{s} + \frac{3}{s+2} - \frac{4}{s+3} \right]$$

$$f(t) = \frac{1}{6} [1 + 3e^{-2t} - 4e^{-3t}] u(t)$$

$$b) \quad F(s) = \frac{K_1}{s} + \frac{K_2}{s+1} + \frac{K_3}{s+2}$$

$$F(s) = \frac{1}{2} \left[\frac{1}{s} - \frac{2}{s+1} + \frac{3}{s+2} \right]$$

$$f(t) = \frac{1}{2} [1 - 2e^{-t} + 3e^{-2t}] u(t)$$

$$K_1 = \frac{1}{(1)(2)} = 1/2$$

$$K_2 = \frac{(-1)^2 - 1 + 1}{(1)(-1+2)} = -1$$

$$K_3 = \frac{(-2)^2 - 2 + 1}{(2)(-2+1)} = \frac{3}{2}$$