

13.14 Given the following functions $F(s)$, find $f(t)$.

$$(a) \quad F(s) = \frac{s^2 + 7s + 12}{(s + 2)(s + 4)(s + 6)}$$

$$(b) \quad F(s) = \frac{(s + 3)(s + 6)}{s(s^2 + 10s + 24)}$$

SOLUTION:

$$a) \quad F(s) = \frac{(s+4)(s+3)}{(s+2)(s+4)(s+6)} = \frac{s+3}{(s+2)(s+6)} = \frac{K_1}{s+2} + \frac{K_2}{s+6}$$

$$K_1 = \frac{-2+3}{-2+6} = \frac{1}{4} \quad K_2 = \frac{-6+3}{-6+2} = \frac{3}{4}$$

$$F(s) = \frac{1}{4} \left[\frac{1}{s+2} + \frac{3}{s+6} \right] \Rightarrow \boxed{f(t) = \frac{1}{4} [e^{-2t} + 3e^{-6t}] u(t)}$$

$$b) \quad F(s) = \frac{(s+3)(s+6)}{s(s+4)(s+6)} = \frac{s+3}{s(s+4)} = \frac{K_1}{s} + \frac{K_2}{s+4}$$

$$K_1 = \frac{3}{4} \quad K_2 = \frac{-4+3}{-4} = +\frac{1}{4}$$

$$F(s) = \frac{1}{4} \left[\frac{3}{s} + \frac{1}{s+4} \right] \Rightarrow \boxed{f(t) = \frac{1}{4} [3 + e^{-4t}] u(t)}$$