

13.23 Given the following functions $F(s)$, find the inverse Laplace transform of each function. **PSV**

$$(a) F(s) = \frac{s+6}{s^2(s+2)}$$

$$(b) F(s) = \frac{s+3}{(s+1)^2(s+3)}$$

SOLUTION:

$$a) F(s) = \frac{K_1}{s^2} + \frac{K_2}{s} + \frac{K_3}{s+2} \quad K_1 = \frac{6}{2} = 3 \quad K_3 = \frac{4}{(2)^2} = 1$$

$$\text{Let } s = -1: F(-1) = \frac{5}{1(-1)} = \frac{3}{1} + \frac{K_2}{(-1)} + \frac{1}{1} \Rightarrow K_2 = -1$$

$$F(s) = \frac{3}{s^2} - \frac{1}{s} + \frac{1}{s+2} \Rightarrow \boxed{f(t) = [3t - 1 + e^{-2t}]u(t)}$$

$$b) F(s) = \frac{1}{(s+1)^2} \quad K_1 = 1$$

$$\text{Let } G(s) = 1/s^2, \quad f(t) = e^{-t}g(t)$$

$$g(t) = t u(t) \Rightarrow \boxed{f(t) = te^{-t}u(t)}$$