

13.39 Find $f(t)$ using convolution if $\mathbf{F}(s)$ is

$$\mathbf{F}(s) = \frac{1}{(s+1)(s+4)}$$

SOLUTION:

$$\text{Let } F_1(s) = \frac{1}{s+1} \Rightarrow f_1(t) = e^{-t}$$

$$F_2(s) = \frac{1}{s+4} \Rightarrow f_2(t) = e^{-4t}$$

$$f(t) = \int_0^t e^{-(t-\lambda)} e^{-4\lambda} d\lambda = e^{-t} \int_0^t e^{-3\lambda} d\lambda = \frac{e^{-t}}{3} e^{-3\lambda} \Big|_0^t$$

$$f(t) = \frac{e^{-t}}{3} [1 - e^{-3t}] u(t) \quad \boxed{f(t) = \frac{1}{3} [e^{-t} - e^{-4t}] u(t)}$$